Minicourse: Introduction to Mathematics of Deep Learning

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Abstract: The goal of this minicourse of four lectures is to introduce basic concepts from deep learning in a rigorous mathematical fashion, e.g introduce mathematical definitions of deep neural networks (DNNs), loss functions, the backpropagation algorithm, etc. We attempt to identify for each concept the simplest setting that minimizes technicalities but still contains the key mathematics. This minicourse follows the upcoming book "Mathematics of Deep Learning: an introduction" by L. Berlyand and P.-E. Jabin. Publisher: *De Gruyter* (to appear).

Lecture 1. *History, general perspective and basic notions of deep learning*

In this lecture, we briefly discuss the general perspective of machine learning:
what is it and why study it? Next, we introduce the classification problem in a supervised learning context and then introduce the key concept of artificial neural networks (ANNs) as the composition of linear maps and nonlinear activation function followed by other basic definitions describing ANNs.

Lecture 2. *DNNs and approximation theory*

In this lecture, we discuss the universal approximation theorem describing the wide class of continuous functions which DNNs can be used to approximate. This theorem explains the extensive use of DNNs in classification problems. Next, we introduce the concept of training via the gradient descent algorithm which improves the approximate classifier by iteratively "learning" from the dataset.

Lecture 3. *Backpropagation & CNNs*

We begin from introducing the notion of computational complexity. Next, we introduce the backpropagation algorithm which significantly reduces the computational cost of optimizing the loss function. This is done in the simplest one neuron per layer setting, which while not practical allows us to explain the concept without many technicalities. Finally, if time permits, we briefly discuss convolutional neural networks and their properties.

Lecture 4. *Implementing DNNs and Training: a brief overview of Pytorch*

This lecture will be presented by my co-author P.-E. Jabin (Penn State).

We present here a short and very basic introduction to Pytorch, in the context of classification of images. We assume passing familiarity with coding and with python in particular. Many further tutorials exist online and several can be found at https://pytorch.org/tutorials/