Hyperbolic Conservation Laws: theory and numerical methods

The course will focus on the class of *Hyperbolic Conservation Laws* and will be split in two major components: *Analysis* and *Numerical* methods.

The objective is to concisely study this type of problems and to gain a first understanding of the dynamics of their solutions. We will do so by highlighting some of the main components of the theory and the numerical methods used to solve them. It is expected that, after this course, the students will be able to navigate through the literature of these problems and to further elaborate in their analytical as well as numerical study.

The course will be structured as follows:

1. Introduction to *Conservation Laws* and derivation from basic principles.
2. *Scalar Conservation Laws*
    a. *Shock and rarefaction waves*
        i. *Rankine-Hugoniot* condition
        ii. *(first) Entropy condition*
        iii. *Lax-Oleinik* formula
    b. Weak solutions
        i. (second) *Entropy condition*
        ii. *Uniqueness* of entropy solutions
    c. *Riemann* problems
        i. Sup-norm decay of solutions
3. Numerical solution of *Scalar Conservation Laws*
    a. Derivative approximations *and the Finite Differences* method
        i. *Upwind*, *Lax-Friedrichs*, and *Lax-Wendroff* schemes
        ii. *Courant-Friedrichs-Lewy* condition and numerical *stability*
        iii. *Truncation error* and m*odified equation analysis*
    b. *Numerical Flux functions and the Finite Volumes method*
4. *Introduction to systems of Conservation Laws*

Previous experience with *Partial Differential Equations, Weak Solutions of PDEs,* or *Numerical Methods for Differential Equations* will be beneficial but not necessary. The course will be, to a large extent, self-consistent.

The lecture will be taught in *English*.

Literature

[A.1] L. E. Evans. *Partial Differential Equations*, American Mathematical Society
[A.2] P. D. Lax. *Hyperbolic Partial Differential Equations*, American Mathematical Society
[N.1] K. W. Morton, D.F. Mayers. *Numerical Solution of Partial Differential Equations*, Cambridge University Press
[N.2] R. J. LeVeque. *Finite-Volume Methods for Hyperbolic Problems*, Cambridge Texts in Applied Mathematics

Lecturer: Dr. Nikolaos Sfakianakis
Time: **Wednesday 11:00-12:30 (SWS** 2**)**
Place: **SR1**
Course webpage: http://www.biostruct.uni-hd.de/Vorlesung_Hyperbolic_Sfakianakis.php

Registration: sfakiana@math.uni-heidelberg.de until **March 30th**